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Primary School Children's Errors in Mathematical Division

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Abstract

Children's performance in mathematics is not at satisfactory level in primary and secondary schools in Sri Lanka. Children's mathematical error is one of the main reason for this low achievement. The main objective of this study is to identify the error patterns made in mathematical division, referred to as 'division' in this paper, by the primary school children. For this purpose, a paper-pencil test was administered to 1100 children from 24 schools in Colombo district, studying in Grades 4 and 5. Each paper consisted of 50 questions. The students' answers were marked by the researcher and the wrong responses were analyzed. 350 children were selected randomly from the 1100 children and interviewed, for an in-depth study in order to identify any error patterns.The errors identified were classified ascareless errors, random errors and systematic errors. Most of the errors identified were systematic errors. Further, the study analyzed the following categories of errors in detail: Recalling error, Direction of operation error, operation sign related error, zero related errors, and concept related errors and incomplete operation errors. A very common error was incomplete operations in division under the category of systematic errors. This study suggests that, teachers have to change their teaching methodologies when they teach mathematics. They have to use the teaching aids (abacus, dines blocks, number cards etc.) to explain the basic concepts of divisions.
Key Words:mathematical division, errors, systematic errors, incomplete errors

Introduction

Contemporary life demands a very good mathematical knowledge.Mathematics isimportant for life and supports all-around personal development. Mathematics significant lyinfluences students'education, both in a special branch (mathematical knowledge) andintermsof moral education (Hodaňová and Nocar 2016) Primary mathematics is the basis for the secondary mathematics. Primary mathematics consists of six main topics – Numbers, Mathematical Operations, Measurement, Money, Space and Shapes and Data Handling. These concepts start at Grade One and develop up to Grade Five in the primary circle (Mukunthan, 2013). The four mathematical operations -

Addition, Subtraction, Multiplication and Division are included under the main topic Mathematical Operations.

All four mathematical operations Addition, Subtraction, Multiplication and Division are introduced at Primary level. A method of distributing a set of items into equal parts is division. Division is an inverse operation of multiplication. The issues of learning calculation methods, especially algorithms for division, has been considered in the literature on account of its cognitive and didactical importance.

Division is a fundamental operation that is closely related to multiplication, and is a necessary skill for understanding fractions and some algebraic concepts. Outside the classroom, children often face division-situations such as exchanging snacks with siblings or breaking into groups for a game.

Students use a range of solution strategies to solve multiplication and division word problems and from this it has been inferred that they acquire various intuitive models of multiplication and division (Fischbein, Deri, Nello, & Merino, 1985; Kouba, 1989; Greer, 1992). Many topics in school mathematics depend on multiplication and division as foundational concepts. Failure of children to understand these concepts in the right perspective creates initial problems. As these difficulties robs them of that necessary solid foundation. They need to build on subsequently. This can even continue to impede their progress (Hurst & Hurrell, 2016) in later years.

Dividing by 2, 3, 4 and 5 using practical methods and dividing two-digit numbers by 2 with or without remainder are in Grade 3 mathematics curriculum.It develops as dividing two or three digit numbers by 2, 3, 4 and 5 with or without remainder in Grade 4 and dividing two or three digit numbers by a number (from 2 to 10) with or without remainder in Grade 5.

The Sri Lankan Primary School curriculum places a strong focus on learning mathematics. The innovative steps taken to improve the quality of education are many. Division is an important concept in mathematics. Low performance in Mathematics is evident not only at the primary level but also at secondary level too. Many children make errors on this concept for several reasons in Sri Lanka.Division is one of the main conceptsin the mathematical operations. Therefore, this study tries to find the children's error patterns in mathematical division.

Literature Review

In different contexts, an error can mean different things.Mathematical error may be described as a mistake in a calculation. In the classroom, students make a variety of mathematical errors.*According to Drews (2005) error could be made due to following reasons. It could be the result of carelessness, misinterpretation of symbols or text; lack of relevant experience or knowledge on the mathematical topic/learning objective/concepts; a lack of awareness or inability to check the answer given; or the result in misconception.*

In mathematics, there are a lot of misconceptions. These misconceptions can appear at any time during a child's education. Some misconceptions take place due to the nature of the child; other misconceptions are the results of the teaching techniques.

According to Mulhern and Greer (1989)

a) *Errors are frequently 'surprising' usually because they may have remained undetected by teacher for some time.*
b) *Errors are often extremely 'persistent'. Typically, these are resistant to change themselves and correction of errors may involve fundamental reorganization of pupil's knowledge.*
c) *Frequently errors 'ignore meaning'. So that an answer which is obviously incorrect is rejected with no questioning.*

Errors can occur in many ways, as described above. According to Brousseau et al (in Mulhern and Greer, 1989) errors may occur in four main ways in mathematics.

a) *As a result of major misconceptions about fundamental aspects of mathematics*
b) *As a result of correct and faithful application of systematically flawed procedure*
c) *When the flawed rules and misconceptions the pupils possess are not recognized by the teacher.*
d) *Due to the use of highly original, non-formal methods of solving problems invented by pupils.*

Mathematical errors are significant in practice because they can be used to enhance teaching and learning. Fiori and Zuccherri (2005) showed that certain error patterns depend on different ethnic-cultural situations, teaching methods and algorithm used, and to what extent they are used.

Many primary mathematics experts analyzed several types of mathematical errors. They have classified errors under few patterns. Cox (1975) classified the mathematical errors into three major categories viz:

(i) ***Careless error**: A student misses one or two problems out of five problems of a given type*

(ii) ***Random error****: A student misses three or more problems out of five problems of a given type, but no pattern is apparent*

(iii)***Systematic error****: A student misses three or more problems out of five problems of a given type, using the same incorrect process as evidenced by the presence of a repeated pattern*

Nesher (1987), used the term misconceptions to describe systematic errors without reference to a theoretical position.

According to Yang (2014) not all errors can be attributed to reasoning faults; some are simply careless errors.Error analysis is concerned with the pervasive errors (or 'bugs') which learners make, based on their lack of conceptual or procedural understanding (KetterlinGeller & Yovanoff 2009).Error analysis, also referred to error pattern analysis, is the study of errors in learners' work with a view to finding explanations for these reasoning errors (Herholdt & Sapire 2014).Error analysis is a process of reviewing the errors with an objective to provide feedback and remediation instructions to improve the learning and performance (Muthukrishnan et al 2019)

Radatz (1979) classified the errors in terms of

(1) Language difficulties. Mathematics is like a "foreign language" for students who need to know and understand mathematical concepts, symbols, and vocabulary. Misunderstanding the semantics of mathematics language may cause students' errors at the beginning of problem solving;

(2) Difficulties in processing iconic and visual representation of mathematical knowledge;

(3) Deficiency in requisite skills, facts, and concepts; for example, students may forget or be unable to recall related information in solving problems;

(4) Incorrect associations or rigidity; that is, negative transfer caused by decoding and encoding information; and

(5) Application of irrelevant rules or strategies.

Department for Education and Employment (1999) of United Kingdom published a model for diagnosing children's error.

(i) *Computational error / Careless mistake :-*

(ii) *Misconceptions*

(iii) *Wrong Operation :-*

(iv) *Over-generalization ; Eg:-* $(x \times 0 = 0)$. but $(x + 0 \neq 0)$

(v) *Under-generalization*

(vi) *Random response*

Error analysis or error pattern analysis is the analysis of errors in learners' mathematical workings with the aim of identifying the common patterns and to find explanations for the causes of these errors (Herholdt and Sapire, 2014).

Fiori and Zuccherri (2005) found that certain error patterns depend on different ethnic-cultural situations, teaching methods and algorithm used, and to what extent.

Methodology and Results

The objective of this study is to identify the patterns of errors made by the students in division.

National Schools, Type 1 AB, Type 1C, Type 2 and Type 3 are the five categories of schools in Sri Lanka. There are no elementary schools under the category National Schools. In these schools, students are taught in Sinhala, Tamil, and English. For this study, Stratified Random Sampling Method was used to pick all schools with primary sections. Twenty four schools were chosen as sample.

The question paper consisted of 50division sums. The Researcher designed the question paper. It was administered to 1100, Grades 4 and 5 students from the 24 sample schools. The duration of the question paper was Two hours. All the sums (1100 X 50= 55,000) were marked by the researcher.Then the researcher sort out the sums with the errors. Then he identified the pattern of errors from those sums. 350 students were randomly selected and interviewed and the reason for their answers were inquired and recorded by the researcher

Only 210 students answered all the questions correctly. The sample answered 36,510 sums out of the 55,000 sums correctly. Therefore 18,490 (36.95%) answers were wrong. There answers were analyzed by the researcher..

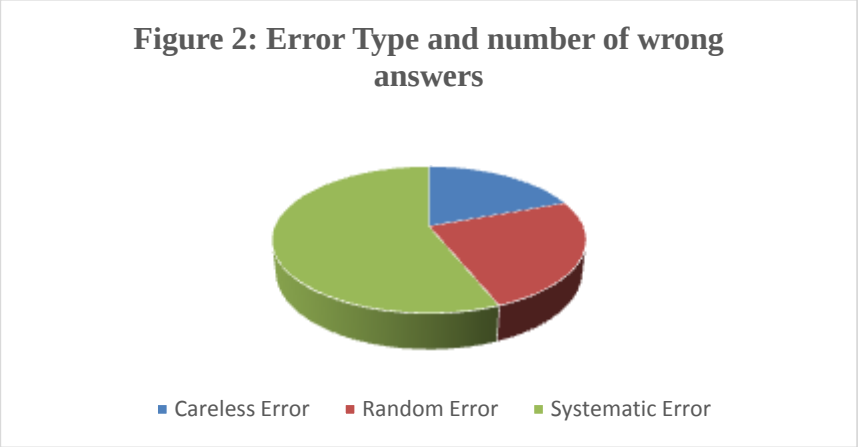
According to Cox *(1975) children's errors were classified as random errors, careless errors and systematic errors. The type of errors and the percentages are given in the Table below:*

Table 1

Error Type and number of wrong answers

Error Type	Number	Percentage
Careless Error	3,587	19.40
Random Error	4,526	24.48
Systematic Error	10,377	56.12
Total	18,490	100.00

The data in above table can be shown as in the graph.



These systematic errors (10377) were further grouped. Yetkin (2003) reported that majority of the students' errors are often systematic and rule-based rather than non-systematic errors. Error analysis focusses on the systematic errors which occurred consistently and prevalent in students' work that reflects lack of knowledge and skills.

Analyzing the Systematic Errors

Educators typically analyze students' mathematical errors with the intent to improve instruction and correct misconceptions (Mastropieri & Scruggs, 2002).According to Nanayakkara (1992) Systematic Errors could have arisen due to many reasons. Data collected during interviews enabled to find the most important reason. Observation of the children's answers and the interview with them enabled the researcher to identify the following tenerror types.

1. Error Type I

Recalling errors: incorrectly remembering multiplication facts.

Examples

Table 2: Examples for Error Type I

E1	E2	E3	E4	E5
16÷2 =7	27÷3=8	32÷4 =7	40÷5 = 9	36÷6 =5

2. Error Type II

Direction of operation : Start the division from the right hand side.

Examples

Table 3: Examples for Error Type II

E 6	E 7	E 8	E 9	E10
$2\overline{)346}$	$3\overline{)126}$	$4\overline{)548}$	$5\overline{)378}$	$3\overline{)379}$

3. Error Type III

Operation sign related: interprets ÷ as ×

Table 4: Examples for Error Type III

E11	E12	E13	E14	E15
6÷2= 12	10÷2=20	9÷3=27	8÷4=32	10÷5=50

4. Error Type IV

Operation sign related: interprets ÷ as +

Table 5: Examples for Error Type IV

E16	E17	E18	E19	E20
6÷2= 8	10÷2=12	9÷3=12	8÷4=12	10÷5=15

5. Error Type V

Operation sign related: interprets ÷ as -

Table 6: Examples for Error Type V

E21	E22	E23	E24	E25
6÷2= 4	10÷2=8	9÷3=6	8÷4=4	10÷5=5

6. Error Type VI

Zero related error: Zero when divided by a number gives the number as answer.

Examples

Table 7: Examples for Error Type VI

E26	E27	E28	E29	E30
0÷3= 3	202÷2= 121	400÷4= 144	60÷3= 23	208÷2= 124

7. Error Type VII

Zero related error: ignore the zero

Examples

Table 8: Examples for Error Type VII

E31	E32	E33	E34	E35
2 | 204 1 2	2 | 620 31	60÷3= 2	400÷4=1	208÷2= 1 4

8. Error Type VIII

Concept related error: Write down the factors, instead of number of factors

Table 9: Examples for Error Type VIII

E36	E37	E38	E39	E40
6÷2= 2, 2, 2	10÷2=2, 2, 2,2,2	9÷3=3, 3, 3	8÷4=4, 4	10÷5=5, 5

9. Error Type IX

Incomplete operation: Ignores the reminder from the tens column

Table 10: Examples for Error Type IX

E41	E42	E43	E44	E45
12 2)34 2 --- 4 4 --- 0	22 3)76 6 --- 6 6 --- 0	11 4)54 4 --- 4 4 --- 0	10 5)70 5 --- 0 0 --- 0	11 6)76 6 --- 6 6 --- 0

10. Error Type X

Incomplete operation: Each digit of the dividend is divide separately by divisor without performing subsequent operations.

Table 11: Examples for Error Type X

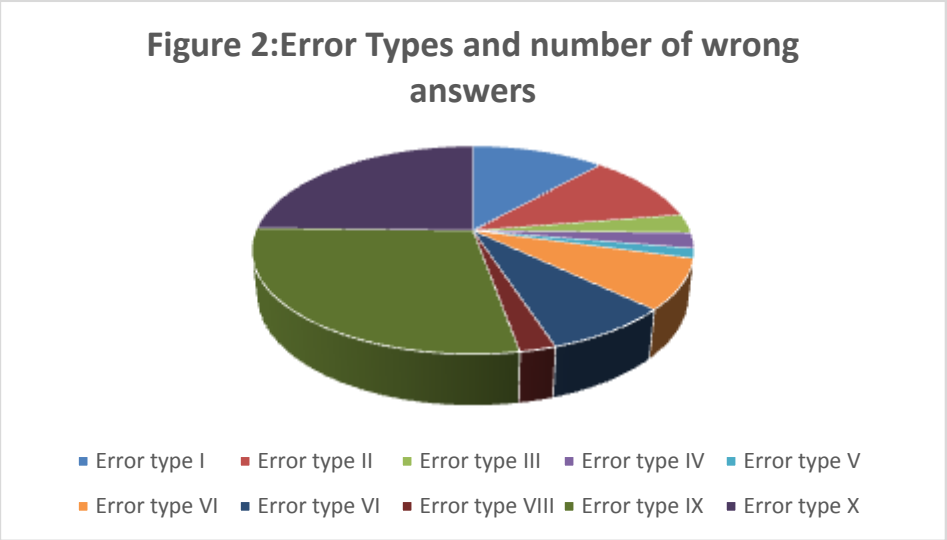
E46	E47	E48	E49	E50
304÷2 = 102	704÷3 = 201	01 4)34 0 --- 4 4 --- 0	10 5)70 5 --- 0 0 --- 0	11 6)76 6 --- 6 6 --- 0

The researcher grouped the children's systematic errors (10,377) according to the error types.

Table 12 : Error Types and number of wrong answers

Error Pattern	Error Type	Number	Percentage
Recalling error	Error type I	1202	11.58
Direction of operation error	Error type II	1123	10.82
Operation sign related errors	Error type III	304	2.93
	Error type IV	250	2.41
	Error type V	156	1.50
Zero related error	Error type VI	810	7.81
	Error type VI	819	7.89
Concept related error	Error type VIII	232	2.24
Incomplete operation errors	Error type IX	2928	28.22
	Error type X	2553	24.60
Total		**10,377**	**100.00**

Above data can be shown as following



The above Table 12 shows the Error Patterns and the Types along with the percentage of students in the sample who made these errors. The Error Patterns consisted of six Types *viz.* Recalling Error, Direction of Operation Error, Operation sign related error, Zero related Error, Concept related error, and incomplete operation error.

Significantly 52.82% of the errors were related to incomplete operation. At the interview, Ramanan, a Grade 5 student told the answer to the question E28:- $2\overline{)34}$= equal to 12. The

researcher asked to Ramanan, ‘ Ramanan, whydid you answer like this ? ‘. He replied that ‘There is one 2 in 3 and 2 twos in four’.

11.58% of the errors were related to recalling errors. Children incorrectly remembering multiplication facts is the reason for this type of errors.

10.82% of the errors were direction of operation errors. Children starts the other three operations (addition, subtraction and multiplication) from the right side. But division starts from the left. Children may generalize the direction starts from right to division too. This may be the reason for this type of errors.

6.84% of the errors were related to operation sign.Khalid & Embong (2020) conducted a study in Malaysia with year 7 children.They noted that “Few students assume that the division sign “÷” is “+” because they look quite similar.”

15.7% of the errors were zero related. The role of zero as a place holder is not well understood. The children didn't know that 0 had to be used to keep the answer's place value.

2.24% of the errors were related to concepts.

The analysis of these error patterns reveal that some of them are general to many types of operations. Following error patterns were seen to be prominent among the children: Starting the operation from a wrong place and proceeding in the wrong direction incorrect recalling of the basic facts and zero related errors.

Conclusion

Present study highlighted that, most of the errors made by the children, related to division were systematic errors. Within the systematic errors incomplete operations were quite significant. Systematic errors, if not arrested at the beginning, may become a serious problem to the learner later in the life as the four major mathematical operations are essential in the day-to-day life of a person.

The Primary Mathematics Textbooks and the Teachers’ Guides give clear instructions to the primary grade teachers on the use of day-to-day life situations in teaching mathematics in Sri Lanka. For example, considering division in a person’s life, it is linked with almost every action of the person.In the context of the children’s school life, many children do

transactions by way of buying sweets or short eats at the school canteen or tuck shop. Here they have to do calculations, especially dividing, when paying for what they have bought. This study suggested that, teachers have to change their teaching methodologies when they teach mathematics. They have to use the teaching aids (abacus, dines blocks, number cards etc.) for explain the basic concepts of divisions.

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